Mathematical Analysis G12MAN 2008/9

Assessed Coursework 1

Due 5 pm on Monday 27 October 2008.

Unless otherwise specified, you must always justify your answers.

Throughout this coursework, d denotes a fixed positive integer.

Where you use standard results, state clearly which they are.

There are hand-in boxes in the Maths/Physics building and in the Pope building. You must fill out a yellow coursework cover sheet, staple it to your work, and date-stamp it using the machine provided, before placing your work in the box. Cover sheets are available next to the hand-in boxes.

Total marks obtainable 100. Each question is worth 25 marks.

1 (a) Illustrate on a diagram the following subsets of $\mathbb{R}$. [You should show your working for parts (c) and (d).]

(i) $A =]-1, 2] \cup [3, 6]$ [3 marks]

(ii) $B = [4, \infty[$ [2 marks]

(iii) $C = \mathbb{Z} \cap]0, 6[$ [4 marks]

(iv) $D = \{x \in \mathbb{R} \mid x^3 > 9x\}$ [8 marks]

(b) Consider your sketches of the sets A to D above. Without further justification, answer the following question. Which of the sets A to D are bounded and which are unbounded? [8 marks]

2 Let A and B be bounded subsets of $\mathbb{R}^d$. Set

$$C = \{\boldsymbol{x} + \boldsymbol{y} \mid \boldsymbol{x} \in A,\ \boldsymbol{y} \in B\}$$

(the set of all vectors in $\mathbb{R}^d$ that can be obtained by adding some element of A to some element of B).

Prove that C is a bounded subset of $\mathbb{R}^d$. [25 marks]

3 (a) Sketch the following subset of $\mathbb{R}^2$, indicating all key features clearly:

$$S = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq y^2 - x^2 < 4\}.$$ [15 marks]

(b) Using your answer to (a) to help you, write down your answers to the following questions without further justification.

(i) Is the set S bounded? [4 marks]

(ii) What is the set $\operatorname{int} S$?

[Your answer should be a **specific** subset of $\mathbb{R}^2$.] [3 marks]

(iii) What is the set $\operatorname{nint} S$?

[Again, your answer should be a **specific** subset of $\mathbb{R}^2$.] [3 marks]

4 Let A and B be subsets of $\mathbb{R}^d$. Is it necessarily true that $\operatorname{int}(A \cap B) = \operatorname{int}(A) \cap \operatorname{int}(B)$?

[Give either a proof, or else a specific counterexample with justification.] [25 marks]