Mathematical Analysis **G12MAN** **2008/9**

Feedback on Assessed Coursework 2

Common errors, pointers for improvement, and other comments

The results on this coursework were mostly good. The feedback below is based on comments passed to me by the postgraduate markers. As last time, this agrees with what I saw on the scripts that I looked through.

1. This question was mostly well done. However, some students lost marks by not mentioning the word 'hyperbola'. Other marks were lost by not showing some of the working used to determine which region of the diagram was needed, or by omitting some labels from the diagram or some other relevant details.

2. Again, this was generally very good. The most common problem was with unclear expression/communication: sometimes students did not explain which assumptions they were making or which technique they were using. In particular, some of you did not explain that you were using (or trying to use) the ratio test to justify your answers.

   As part of your justification here, you should quote, from lectures, the fact that convergence in $\mathbb{R}^d$ is 'coordinatewise convergence'.

   Some students did not use the ratio test, but simply claimed that the denominator got bigger much faster than the numerator. This was not convincing.

3. You mostly did well on this question too.

   (a) Some students believed that this function was continuous. In some cases these students tried to use the Sandwich Theorem, but ended up with 0 on one side of the sandwich and ∞ on the other. This would not help: you need the same number on both sides of your sandwich.

   Some students appeared to mix two different methods to show that the function was discontinuous at $(0,0)$. You need to establish that it is **NOT** true that $f(x,y) \to f(0,0) = 0$ as $(x,y) \to (0,0)$. One way is to show that the limit does not exist **at all**: for example, you could find two different ways to approach $(0,0)$ along which the limits are different. Alternatively, you can find **one** direction of approach along which the function does not tend to 0: **that is already enough to disprove** $\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

   (b) Most people made good use of the Sandwich Theorem to prove the continuity of this function at the point $(0,0)$. However, quite a few students lost marks by forgetting to explain why the function was continuous at the other points of $\mathbb{R}^2$.

4. (a) Here many students defined a function whose values depend on n. However there is only supposed to be **one** function f here from $\mathbb{R}$ to $\mathbb{R}$, and you can't define the function differently for each pair of points x_n and y_n. Indeed, there are many problems with this, but at the very least you would need to explain what your function f does to the remaining points of $\mathbb{R}$.

   Some students tried to use the function $f(x) = 1/x$. However, this does not satisfy the conditions of the question: your function f must be defined and continuous at **ALL** points of $\mathbb{R}$. Related to this, many students tried a pair of bounded sequences (x_n) and (y_n), ignoring the hint given in the question (which pointed out that this could not work).

   Many of the answers were unclearly expressed, both in the use of English and in the assumptions made: people did not make adequate use of words such as 'let', 'assume', 'then', 'hence', 'thus', etc.; some students tried to manipulate ∞ in ways that did not make sense, claiming, for example, that $\infty - \infty = 0$.

(b) You found this part rather hard. As you can see from the model solution, you need to apply the definition of sequential compactness to **one** of the two sequences involved. You do not need to apply it to both sequences, and if you do it can lead to problems, because you would still need to explain why you think that you can use the **same** choice of $n_1 < n_2 < n_3 \cdots$ for **both** sequences.

The most common error was to state that both sequences must actually converge already. In some cases, people's claims appeared to contradict the example from part(a). But even under the conditions of (b), the original sequences $(\boldsymbol{x}_n)$ and $(\boldsymbol{y}_n)$ need not converge. (As an **exercise**, you can think of some examples based on the divergent sequence of real numbers $((-1)^n)_{n=1}^{\infty}$, or more interesting examples such as $(\sin(n))_{n=1}^{\infty}$.)

Many students wrote that $\boldsymbol{x}_n \to \boldsymbol{y}_n$ as $n \to \infty$. This does not make sense. The limit (if it exists) of a sequence $(\boldsymbol{x}_n)$ must not depend on n. Again, many of these attempted arguments appeared to contradict the result from (a).

Some students did not state the definition of sequential compactness correctly, and so were not able to use this definition properly.