Mathematical Analysis **G12MAN** **2009/10**

Feedback on Assessed Coursework 1

Common errors, pointers for improvement, and other comments

Overall, the results on this coursework were good. The feedback below is based on a combination of comments passed to me by the postgraduate markers and what I saw on the scripts that I looked through.

1. (a) This part was mostly well done. There were some slips with the endpoints of intervals, and occasionally one or two features of these sets (such as their infinite extent) was unclear. Also, sometimes some of the working in identifying sets C and D was missing, unclear or inaccurate. The most common mistakes for set C were: claiming that $n^2 \leq 7 \Leftrightarrow n \leq \sqrt{7}$ (it should be $|n| \leq \sqrt{7}$); sketching a continuous **interval** for C instead of a discrete set containing precisely five points. For D, some students lost half of the set by not being careful enough. The safest method here is to use a sign chart, as in the solutions.

   (b) This part was mostly very well done, with answers matching the sketches from part (a).

2. This question was quite well done. Most of you correctly used the information in the question to obtain appropriate bounds for the modulus/norm of the elements of the sets A and C, and correctly multiplied these bounds together to obtain an appropriate bound for D.

   There were some problems with expression.

   In order to prove that every element of D has a certain property, it is safest to start with

   'Suppose that $\boldsymbol{y} \in D$',

   and then make deductions about $\boldsymbol{y}$.

   Some students treated elements of A as vectors, used norm notation instead of modulus, and appeared to multiply two vectors together during the proof. In this question, you should make it very clear that you are using scalar multiplication, and multiplying real numbers in A by vectors in C. Otherwise the argument looks very confusing, and is unconvincing.

3. (a) Many students lost marks by not including enough detail/explanation/justification here. In particular, many students did not include as many labels as they should, and did not indicate that the regions extended off to infinity (using dotted lines). This also led to lost marks in part (b) if the diagram did not make it sufficiently clear that the set was unbounded. Some students 'lost' half of the set S by not being careful enough with the justification, or had bands which turned round a corner.

   (b) Most students did very well on this part. In particular, I was pleased to see a lot of careful use of set notation. As mentioned above, further marks were lost here if it was not sufficiently clear from the sketch in (a) whether the set S was bounded or not.

4. (a) There was some very good work here. However, there were some quite instructive problems with quantifiers, expression, logic and proof structure.

Some students attempted to prove this result using a diagram. Unfortunately, in this question, the diagram needs to be backed up by more formal reasoning: a convincing 2-dimensional diagram might not be valid in the general setting of $\mathbb{R}^d$.

As some students are confused about the definition of nint, you are recommended to include the definition in your solution in order for it to be fully convincing. For example, some students treated $\operatorname{nint} A$ as if it was the complement of $\operatorname{int} A$ in $\mathbb{R}^d$, instead of the correct definition, $A \setminus \operatorname{int} A$. It was unclear throughout whether this was a genuine misunderstanding of the definition of nint, or whether it was a problem with the expression of the proof. Other students actually used the definition of $\operatorname{int} A$, and claimed this was the definition of $\operatorname{nint} A$.

Many students correctly noted that $\operatorname{nint} A \cap \operatorname{nint} B \subseteq A \cap B$ (though with some problems of the above type). The main problems came with showing that

$$(\operatorname{nint} A \cap \operatorname{nint} B) \cap \operatorname{int}(A \cap B) = \emptyset$$

(or the equivalent).

Students often started correctly with something like 'Suppose that $\boldsymbol{x} \in \operatorname{nint} A \cap \operatorname{nint} B$'. (Note that 'Suppose' is a bit safer than 'Let' here). Some students then failed to explain convincingly why $\boldsymbol{x}$ had to be in $A \cap B$. But the main problem came with showing that $\boldsymbol{x} \notin \operatorname{int}(A \cap B)$. One way to do this is given in the solutions. However, if you want to work directly from the definitions, you need to establish that, for all $r > 0$, $B_r(\boldsymbol{x})$ is not a subset of $A \cap B$. This means that you should say 'Let $r > 0$' (or 'Suppose that $r > 0$'), and make deductions from that. However, many students wrote something like the following:

'Since $\boldsymbol{x} \in \operatorname{nint} A$, for all $r_1 > 0$, $B_{r_1}(\boldsymbol{x})$ is not a subset of A.
Since $\boldsymbol{x} \in \operatorname{nint} B$, for all $r_2 > 0$, $B_{r_2}(\boldsymbol{x})$ is not a subset of B.
Set $r = \min\{r_1, r_2\} > 0$. Then $B_r(\boldsymbol{x})$ is not a subset of $A \cap B$, and so $\boldsymbol{x} \notin \operatorname{int}(A \cap B)$.'

There are several problems with this. First of all, we never saw the all-important 'Let $r > 0$' (or equivalent) that we need to establish that something is true for all r. Secondly, it does not make sense to follow the first two statements ('for all $r_1 > 0 \ldots$' and 'for all $r_2 > 0 \ldots$) with 'Set $r = \min\{r_1, r_2\} > 0$.' This last statement might be appropriate if we had previously said 'There exist $r_1 > 0$ and $r_2 > 0$ such that ...' (or equivalent). It is an instructive exercise for you to think about the difference in these settings.

(b) This was mostly well done. However, quite a few students failed to give **specific** examples (as requested in the question), and attempted to draw non-specific generic diagrams of a typical 2-dimensional situation. Note that you can do quite a lot with diagrams if you specify exactly which sets you are working with, but otherwise the diagrams will not be fully convincing.

Some students gave two specific sets A and B, but then miscalculated $A \cap B$, thus spoiling their answer. Indeed, some made unlucky choices of A and B for which equality **did** hold.

Nevertheless, in most cases it was clear that people had a good idea of what the two sets under discussion were, and why they were likely to be different