**Mathematical Analysis** **G12MAN** **2009/10**

## Assessed Coursework 2

Due 5 pm on Thursday 26 November 2009.

Unless otherwise specified, you must always justify your answers.

Throughout this coursework, $d$ denotes a fixed positive integer.

Where you use standard results, state clearly which they are.

There are hand-in boxes in the Maths/Physics building and in the Pope building. You must fill out a yellow coursework cover sheet, staple it to your work, and date-stamp it using the machine provided, before placing your work in the box. Cover sheets are available next to the hand-in boxes.

Total marks obtainable 100. Each question is worth 25 marks.

**1** Consider the following subset of $\mathbb{R}^2$:

$$S = \{(x, y) \in \mathbb{R}^2 \mid (x+4)^2 + y^2 > 25 \text{ and } (x-4)^2 + y^2 \leq 25\} .$$

(a) Draw a carefully labelled sketch of the set $S$. You should show your working, describe the key features of the set $S$, and indicate these key features clearly on your sketch. [16]

(b) Using your answer to (a) to help you, write down your answers to the following questions without further justification.

**Your answers should match the sketch from part (a).**

(i) Is the set $S$ bounded? [3]

(ii) Is the set $S$ open? [3]

(iii) Is the set $S$ closed? [3]

**2** Let $A \subseteq \mathbb{R}^d$, and set $E = A \cup \operatorname{nint}(A^c)$.

(a) Prove that $E$ is a closed subset of $\mathbb{R}^d$. [10 marks]

(b) With $A$ and $E$ as above, let $F$ be a closed subset of $\mathbb{R}^d$, and suppose that $A \subseteq F$. Prove that $E \subseteq F$. [15 marks]

**3** Consider the following four subsets of $\mathbb{R}$:

$$A = [1,3] \cup ]3,5]\,; \quad B = [2,5] \setminus \mathbb{Q}\,; \quad C = [2,5] \setminus \mathbb{N}\,; \quad D = ]-\infty, 1]\,.$$

(a) Write down, **without justification**, what the sets of non-interior points in these sets are, i.e., what are $\operatorname{nint} A$, $\operatorname{nint} B$, $\operatorname{nint} C$ and $\operatorname{nint} D$?
Your answers should be **specific** subsets of $\mathbb{R}$. [6]

(b) Which, if any, of these sets are open? [**Your answers should match your answers to part (a)**, but no further justification is required.] [4]

(c) Which, if any, of these sets are closed? [No justification required.] [4]

(d) Which, if any, of these sets are bounded? [No justification required.] [4]

(e) Which, if any, of these sets are sequentially compact? [Here you should justify your answers **by quoting a suitable result from the module**, and using your answers to parts (c) and (d) above. **Your answers here should match your answers to (c) and (d) above**.] [7]

**4** Determine, with justification, whether or not the following functions from $\mathbb{R}^2$ to $\mathbb{R}$ are continuous.

(a) $$f(x,y) = \begin{cases} \dfrac{x^6 y^{20}}{x^{16} + y^{32}} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$ [10 marks]

(b) $$g(x,y) = \begin{cases} \dfrac{x^6 y^{22}}{x^{16} + y^{32}} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$ [15 marks]