Mathematical Analysis **G12MAN** **2009/10**

Assessed Coursework 1

Due 5 pm on Thursday 29 October 2009.

Unless otherwise specified, you must always justify your answers.

Throughout this coursework, d denotes a fixed positive integer.

Where you use standard results, state clearly which they are.

There are hand-in boxes in the Maths/Physics building and in the Pope building. You must fill out a yellow coursework cover sheet, staple it to your work, and date-stamp it using the machine provided, before placing your work in the box. Cover sheets are available next to the hand-in boxes.

Total marks obtainable 100. Each question is worth 25 marks.

1 (a) Draw sketches to illustrate the following subsets of $\mathbb{R}$.
You should show your working for parts (iii) and (iv).

(i) $A = [-3, 1[\cup]2, 4]$ [3 marks]

(ii) $B =]-\infty, 1[$ [2 marks]

(iii) $C = \{n \in \mathbb{Z} \mid n^2 \leq 7\}$ [4 marks]

(iv) $D = \{y \in \mathbb{R} \mid y^4 \geq 2y^3\}$ [8 marks]

(b) Consider your sketches of the sets A to D above. Without further justification, answer the following question. Which of the sets A to D are bounded and which are unbounded?
Your answers should match the sketches from part (a). [8 marks]

2 Let A be a bounded subset of $\mathbb{R}$, and let C be a bounded subset of $\mathbb{R}^d$. Set

$$D = \{\lambda \boldsymbol{x} \mid \lambda \in A, \boldsymbol{x} \in C\}$$

(the set of all vectors in $\mathbb{R}^d$ that can be obtained by multiplying some vector in C by some real number in A).

Prove that D is a bounded subset of $\mathbb{R}^d$. [25 marks]

3 Consider the following subset of $\mathbb{R}^2$:

$$S = \{(x, y) \in \mathbb{R}^2 \mid 1 < |y - x| \leq 2\}\,.$$

(a) Draw a carefully labelled sketch of the set S.
You should show your working, describe the key features of the set, and indicate these key features clearly on your sketch. [15 marks]

(b) Using your answer to (a) to help you, write down your answers to the following questions without further justification.

(i) Is the set S bounded? [4 marks]

(ii) What is the set $\operatorname{int} S$?
[Your answer should be a **specific** subset of $\mathbb{R}^2$.] [3 marks]

(iii) What is the set $\operatorname{nint} S$?
[Again, your answer should be a **specific** subset of $\mathbb{R}^2$.] [3 marks]

4 Let A and B be subsets of $\mathbb{R}^d$.

(a) Prove that $\operatorname{nint}(A) \cap \operatorname{nint}(B) \subseteq \operatorname{nint}(A \cap B)$. [15 marks]

(b) Is it necessarily true that $\operatorname{nint}(A) \cap \operatorname{nint}(B) = \operatorname{nint}(A \cap B)$?
You should justify your answer to (b) in one of the following standard ways.
If you believe that the statement is always true, then give a proof.
If you believe that the statement is not necessarily true, then give a **specific** counterexample, with justification. If you give convincing sketches of all of the relevant sets, then the rest of your justification can be brief. [10 marks]