Mathematical Analysis **G12MAN** **2007/8**

Feedback on Assessed Coursework 2

The marks for this coursework were not, overall, as good as the first. Nevertheless, I was still pleased to see that most people had a good understanding of the concepts involved and were able to work with them. Many of you produced excellent work in at least some of the questions.

1. (a) Was mostly very well done, though sometimes some of the detail for A^c was missing.

   (b) This part caused some problems. Many of you were unclear about which points were actually in the set B or precisely what the set B^c was (see solutions for more details). As a result, it was easy for you to miss the fact that 1 is a non-interior point of B^c.

   (c) This part was mostly well done. Some of the sketches did not include labels/descriptions of all the key features. Some mistakes were made in determining where the relevant curves crossed the axes. But generally the sketches were good and the conclusions were correct.

2. This question caused the most problems. Indeed, a fair number of students did not attempt this question at all.

   The implication (a)⇒(b) is relatively easy, as it follows quickly from the Bolzano–Weierstrass Theorem. You just have to state your assumptions carefully, quote the theorem correctly, and then deduce the result in a couple of lines. However, some students tried to PROVE the Bolzano–Weierstrass Theorem here. These proofs were sometimes incomplete or incorrect, and often only worked in $\mathbb{R}$ rather than in $\mathbb{R}^d$. Also, the required deduction of the result in question from the Bolzano–Weierstrass Theorem was sometimes missing.

   The implication (b)⇒(a) is harder, but is similar to part of the proof of the Heine–Borel Theorem from lectures.

   Some students did not appear to appreciate that to prove ¬(a)⇒¬(b) requires you to show that (b) fails for **ALL** unbounded sets. In several case, students gave specific examples. But this approach can only show that (b) fails for **SOME** unbounded sets.

   Those students who imitated the method of proof of the Heine–Borel theorem from lectures generally did well here. The key difference in the proofs is that, in lectures, we only had to prove that no subsequence converges **to a point of E**. In this question, you have to prove that no subsequence converges to a point of $\mathbb{R}^d$ (not necessarily in E), which is a **stronger** statement. Fortunately, the method of proof from lectures works, as long as you change a few words in appropriate places.

3. Note that you were asked for **specific** examples here. Thus you should **not** include any unspecified constants in your answers. (a) and (b) Most of the examples here were good, but many students forgot to quote the Heine-Borel theorem to justify their answers. Remember, you need to make it clear **which** standard results from the module you are using to justify your answers! See the model solutions for an idea of the level of justification expected when you are asked for ‘brief justification’ of your answers.

   Some students made errors with set notation which made it hard to determine which set was intended. This is very risky! Unless I am fairly certain which set you intended, **I can not give much credit for a set which is not correctly specified.**

(c) This was mostly good, but some students gave a **set** instead of a **sequence**. Make sure that you distinguish between sets and sequences.

(d) This was mostly good, though a few extra comments on the details would have helped in many cases.

(e) This was mostly well done. However, some students gave sequences in $\mathbb{R}$, when the question asks for a sequence in $\mathbb{R}^2$.

4. (a) This was well done. A few marks were lost by failing to specify that you **should let** x **tend to** 0 when looking at $f(x, x^4)$ as one way of approaching the origin in $\mathbb{R}^2$ (through points of the form (x, x^4) with $x \neq 0$)). Also, you should choose **either** to demonstrate that the function **has no limit** as $(x, y) \to (0, 0)$, **or** demonstrate that it is NOT true that the limit exists and is equal to $f(0, 0) = 0$. Some students gave a mixture of these two, without clarifying their logic.

   (b) Many of you did this well. Some of you forgot to check continuity away from the origin. then, when checking continuity at the origin, some of you lost marks by not specifying **before** the case by case analysis that $(x, y) \neq (0, 0)$ here. Some students had problems with potential division by zero, having not been careful enough in the analysis. The application of the sandwich theorem was generally good.