Mathematical Analysis **G12MAN** **2007/8**

Assessed Coursework 1

Due 5pm on Thursday 1 November 2007.

Unless otherwise specified, you must always justify your answers.

There are hand-in boxes in the Maths/Physics building and in the Pope building. You must fill out a coursework cover sheet, staple it to your work, and date-stamp it using the machine provided, before placing your work in the box. Cover sheets are available next to the hand-in boxes.

Total marks obtainable 100. Each question is worth 20 marks

1. Illustrate on a diagram the following subsets of $\mathbb{R}$. [You should show your working for part (d).]

   (a) $A = [0, 1[\cup]3, 5[$; [4 marks]

   (b) $B =]-\infty, 2]$; [4 marks]

   (c) $C = \mathbb{Z}$; [4 marks]

   (d) $D = \{x \in \mathbb{R} \big| x^4 \leq 8x\}$. [8 marks]

2. Write down without justification the answer to the following question. Which of the sets A to D in Question 1 are bounded, and which of them are unbounded? [20 marks]

3. Suppose that A and B are bounded subsets of $\mathbb{R}^d$. Prove that $A \cup B$ is a bounded subset of $\mathbb{R}^d$. [20 marks]

4. (a) Sketch the following subset of $\mathbb{R}^2$:

   $$S = \left\{(x, y) \in \mathbb{R}^2 \big| 1 \leq xy \leq 4\right\}.$$ [10 marks]

   (b) Using your answer to (a) to help you, write down your answers to the following questions without justification.

   (i) Is the set S bounded? [4 marks]

   (ii) Which points of S, if any, are interior points of S? [3 marks]

   (iii) Which points of S, if any, are non-interior points of S? [3 marks]

5. Let A and B be subsets of $\mathbb{R}$. Is it necessarily true that

   $$\mathrm{int}(A \cup B) = \mathrm{int}(A) \cup \mathrm{int}(B)\,?$$

   [Give a proof or an explicit counterexample.] [20 marks]