Mathematical Analysis **G12MAN** **2007/8**

Feedback on Assessed Coursework 1

Overall, the results on this coursework were excellent. I was very pleased to see that people had a good understanding of the concepts involved and were able to work with them.

1. This question was mostly well done. Occasionally one or two features of these sets (such as their infinite extent) was unclear. Also, some of the working in identifying set D was either missing, unclear or inaccurate.

2. The marks on this question were very high. I have had some comments that to obtain 20 marks for just four words seems excessive. However, they do have to be the **right** four words: if this ensured that people have really made sure that they understand what the term bounded means, then I think this question has achieved its purpose!

3. A large number of students produced perfect or near-perfect proofs based directly on the definition of the term bounded, as in the model solutions. (The model solutions are available from the module web page, in case you missed the relevant announcement.)

   Unfortunately, some students attempted to use a proposition from the notes which showed that boundedness was equivalent to various other conditions (such as being contained in some ball centred on an arbitrary point of $\mathbb{R}^d$). These other conditions are harder to work with, and these attempted proofs were more complicated and not universally successful.

   Another big problem was when students assumed from the beginning, without justification, that the sets A and B were both contained in a ball (centred on the origin) of the **same** radius, without justification. Unfortunately, this assumption is equivalent to the result that you were asked to prove, and so little credit could be awarded in these cases. It was vital here to assume initially that they might need different radii, and THEN to deduce that there is one radius that works for both sets at the same time, and hence for their union.

4. (a) Most of the sketches here were good, though some did not clearly indicate all of the key features. For example, not everyone labelled the relevant curves; a few key points on the curves should be shown; you should clarify in writing that the region you want is shaded (or indicated by appropriate means); preferably you should ensure that the reader knows that you know (!) that these boundary curves are included in the region. I would add this last part in writing. (This is not so much of an issue when you have a mixture of dotted curves and full curves in the sketch, as it is then clear that you are distinguishing between included and excluded boundary curves.)

   (b) This was mostly well done, though some had difficulty using set notation. Make sure that you know how to write down the definition of a set clearly and correctly (see solutions).

5. Most students realized that this was false and attempted to give a counterexample. Most of the counterexamples were fine. Some students were 'unlucky' in their choice of sets: little credit could be awarded where the sets A and B specified in the solution actually satisfied

   $$\operatorname{int}(A \cup B) = \operatorname{int} A \cup \operatorname{int} B\,.$$

   Some students attempted to specify the interiors of the sets instead of specifying the sets themselves, e.g. 'Suppose that $\operatorname{int} A = (0, 1)$ and $\operatorname{int} B = (1, 2)$. Then . . .'. This is not acceptable, especially here, as if the sets are equal to their interiors then you can not obtain a counterexample. But in any case, for a **specific** counterexample, you must **fully specify** what the sets are, not just say 'any sets with the following properties will do'.

Some students appeared to define subsets of $\mathbb{R}^2$ instead of $\mathbb{R}$, or failed to correctly define sets at all. For example, **the following does NOT define any sort of set**:

$$\{x, y \in \mathbb{R} : |x - y| \leq 3\}.$$

This looks like it could be intended to mean

$$\{(x, y) \in \mathbb{R}^2 : |x - y| \leq 3\}.$$

But equally it could be an attempt to define some sort of subset of $\mathbb{R}$. Unfortunately this expression is meaningless. I do not know which set is intended here.

If you are asked to give an example of a set, you **MUST** use notation that makes sense. **Otherwise, little credit can be given.**